

KUNS-1457
HE(TH) 97/11
hep-th/9706144

Born-Infeld Action and Chern-Simons Term from Kaluza-Klein Monopole in M-theory

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Abstract

We investigate the zero modes of the Kaluza-Klein monopole in M-theory and show that the Born-Infeld action and the Chern-Simons term of a D6-brane are reproduced to quadratic order in the field strength of the $U(1)$ field on the brane.

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[†]Supported in part by Grant-in-Aid for Scientific Research from Ministry of Education, Science and Culture (#5416).

1 Introduction

M-theory [1] is a powerful tool for understanding various results in string theory. Although only its low energy effective theory is known, if we believe it, we can derive quantities in string theory without carrying out stringy calculations [2, 3, 4].

In string theory, it is known that the D p -brane is governed by the following action, which is the sum of the Born-Infeld action and the Chern-Simons action (in present paper, we ignore fermion terms):

$$S_{Dp} = S_{BI} + S_{CS}, \quad (1)$$

$$S_{BI} = \frac{e^{-\phi_0}}{(2\pi)^p l_s^{p+1}} \int d^{p+1}\sigma \det^{\frac{1}{2}} \left[-G_{ij} + (2\pi l_s^2) V_{ij} \right], \quad (2)$$

$$S_{CS} = \sqrt{2\pi} (2\pi l_s)^{3-p} \int_{p+1} C e^{(2\pi l_s^2) V}, \quad (3)$$

$$V_{ij} = \nabla_i V_j - \nabla_j V_i - \frac{1}{2\pi l_s^2} B_{ij}, \quad (4)$$

where G_{ij} and B_{ij} are the pull-backs to the D-brane world volume of the spacetime metric G_{MN} and the NS-NS two form field B_{MN} . C is the sum of the R-R 1, 3, 5 and 7-form fields. $V = (1/2)V_{ij}d\sigma^i \wedge d\sigma^j$ is the field strength of the $U(1)$ gauge field V_i living on the D p -brane shifted by B_{ij} . V_i is normalized so that the charge of the string endpoint is one. $l_s = \sqrt{\alpha'}$ is the string length scale.

For $p = 2$ and 4 , S_{Dp} can be derived from the actions of M2 and M5-brane in M-theory [4], and for $p = 0$, it is obtained as the effective action of the Kaluza-Klein mode of the gravity multiplet in eleven dimensional supergravity. For $p = 6$, it has been expected that the M-theory configuration corresponding to D6-brane is a Kaluza-Klein monopole [5]. Kaluza-Klein monopole is a Dirac monopole of $U(1)$ gauge field which appears when a higher dimensional theory is compactified. From the viewpoint of the higher dimensional theory the monopole is a gravitational instanton.

The purpose of this paper is to show that the Born-Infeld action (2) and the Chern-Simons one (3) on a D6-brane are reproduced as the effective action of the zero modes on the Kaluza-Klein monopole. Let us take the static gauge $\sigma^i = y^i$. Then (2) and (3) are expanded as follows (we put $p = 6$ and neglect higher order terms of the Born-Infeld action),

$$\begin{aligned} S_{BI} &= -\frac{e^{-\phi_0}}{(2\pi)^6 l_s^7} \int d^7y \left[\frac{1}{2} (\partial_i \phi^a)^2 + \frac{1}{4(2\pi l_s^2)^2} V_{ij}^2 \right], \\ S_{CS} &= \frac{\sqrt{2\pi}}{(2\pi l_s)^3} \int d^7y \epsilon^{i_1 \dots i_7} \left[\frac{1}{7!} C_{i_1 \dots i_7} + \frac{(2\pi l_s^2)}{5! \cdot 2} C_{i_1 \dots i_5} V_{i_6 i_7} \right] \end{aligned} \quad (5)$$

$$+ \frac{(2\pi l_s^2)^2}{3! \cdot 2 \cdot 2 \cdot 2} C_{i_1 i_2 i_3} V_{i_4 i_5} V_{i_6 i_7} + \frac{(2\pi l_s^2)^3}{3! \cdot 2 \cdot 2 \cdot 2} C_{i_1} V_{i_2 i_3} V_{i_4 i_5} V_{i_6 i_7} \Big]. \quad (6)$$

We shall show that the V^2 term in (5) and the first three terms in (6) are correctly reproduced from the Kaluza-Klein monopole.

Our conventions are similar to the ones in [3]. Instead of explaining them, we present the actions of the eleven and ten dimensional theories:

- M-theory

$$S_{M-\text{kin}} = - \int d^{11}x \left[\frac{1}{2\kappa_{11}^2} \sqrt{-g} R + \frac{1}{2 \cdot 4!} \sqrt{-g} K_{KLMN} K^{KLMN} \right], \quad (7)$$

$$S_{M-\text{CS}} = - \frac{\sqrt{2\kappa_{11}^2}}{3!3!4!4!} \int d^{11}x \epsilon^{ABCDEFGH IJK} A_{ABC} K_{DEFG} K_{HIJK}, \quad (8)$$

$$S_{M2} = \frac{g_{M2}}{3!} \int_{M2} dx^L \wedge dx^M \wedge dx^N A_{LMN}, \quad (9)$$

$$S_{M5} = \frac{g_{M5}}{5!} \int_{M5} dx^{N_1} \wedge \dots \wedge dx^{N_6} A_{N_1 \dots N_6}, \quad (10)$$

$$K_{KLMN} = \nabla_K A_{LMN} - \nabla_L A_{MNK} + \nabla_M A_{NKL} - \nabla_N A_{KLM}, \quad (11)$$

$$2\kappa_{11}^2 = (2\pi)^8 l_s^9, \quad g_{M2} = (2\pi)^2 l_s^{\frac{3}{2}}, \quad g_{M5} = (2\pi)^{-1} l_s^{-\frac{3}{2}}. \quad (12)$$

- Type IIA theory

$$S_{NS-NS} = - \int d^{10}x \sqrt{-g} \frac{1}{2\kappa_{10}^2} \left[R - 4(\partial_M \phi)^2 + \frac{1}{2 \cdot 3!} (H_{LMN})^2 \right], \quad (13)$$

$$S_{R-R} = - \int d^{10}x \sqrt{-g} \left[\frac{1}{4} (F_{MN})^2 + \frac{1}{2 \cdot 4!} (F_{KLMN})^2 \right], \quad (14)$$

$$H_{LMN} = \nabla_L B_{MN} + \nabla_M B_{NL} + \nabla_N B_{LM}, \quad (15)$$

$$F_{MN} = \nabla_M C_N - \nabla_N C_M, \quad (16)$$

$$F_{KLMN} = \nabla_K C_{LMN} - \nabla_L C_{MNK} + \nabla_M C_{NKL} - \nabla_N C_{KLM}, \quad (17)$$

$$2\kappa_{10}^2 = (2\pi)^7 l_s^8 e^{2\phi}. \quad (18)$$

2 Taub-NUT manifold

Taub-NUT manifold is a kind of four dimensional gravitational instanton. Far from the center of the instanton, the structure of the manifold is roughly $R^3 \times S^1$. The S^1 bundle over $S^2 \subset R^3$

enclosing the instanton has Pontryagin index one and is topologically equivalent to S^3 . Taking the spherical coordinate (r, θ, ϕ) for R^3 and parameterizing the fiber S^1 by ψ , the metric of the manifold reads

$$ds_{\text{Taub-NUT}}^2 = \frac{r+m}{r-m} dr^2 + (r^2 - m^2)(d\theta^2 + \sin^2 \theta d\phi^2) + (4m)^2 \frac{r-m}{r+m} \left(d\psi + \frac{1}{2} \cos \theta d\phi \right)^2. \quad (19)$$

This metric has one scale parameter m , representing the size of S^1 fiber at $r \rightarrow \infty$. The ranges of the coordinates are

$$m \leq r, \quad 0 \leq \theta \leq \pi, \quad 0 \leq \phi < 2\pi, \quad 0 \leq \psi < 2\pi. \quad (20)$$

The center of the instanton corresponds to $r = m$. This manifold is homeomorphic to R^4 .

Taub-NUT manifold is not only Ricci flat but also hyper Kähler, so that its holonomy group is $SU(2)_L \subset SU(2)_L \times SU(2)_R$. Therefore, half of the (global) 32 supersymmetries corresponding to left handed spinor on the manifold is broken, while the unbroken 16 supersymmetries corresponds to supersymmetries on the D6-brane.

Let us construct a Kaluza-Klein monopole solution corresponding to a D6-brane in type IIA theory with string coupling e^{ϕ_0} . The boundary conditions of type IIA supergravity are

$$\phi(r \rightarrow \infty) = \phi_0, \quad g_{\mu\nu}^{(10)}(r \rightarrow \infty) = \eta_{\mu\nu}. \quad (21)$$

Since the metrics in ten and eleven dimensions are related by

$$g_{\mu\nu}^{(11)} = e^{-\frac{2}{3}\phi} g_{\mu\nu}^{(10)}, \quad g_{11,11}^{(11)} = l_s^2 e^{\frac{4}{3}\phi}, \quad 0 \leq x^{11} < 2\pi, \quad (22)$$

(21) implies the following boundary conditions for the eleven dimensional metric:

$$g_{\mu\nu}^{(11)}(r \rightarrow \infty) = e^{-\frac{2}{3}\phi_0} \eta_{\mu\nu}, \quad (23)$$

$$g_{11,11}^{(11)}(r \rightarrow \infty) = l_s^2 e^{\frac{4}{3}\phi_0}. \quad (24)$$

Using the Taub-NUT solution (19), the metric satisfying the condition (23) is immediately given as

$$ds^2 = e^{-\frac{2}{3}\phi_0} \left[\eta_{ij} dy^i dy^j + ds_{\text{Taub-NUT}}^2 \right], \quad (25)$$

where y^i ($i = 0, \dots, 6$) is the flat coordinate on R^7 parallel to the D6-brane. From the second condition (24), the scale parameter m is fixed as

$$4m = l_s e^{\phi_0}. \quad (26)$$

3 $U(1)$ field on Kaluza-Klein monopole

Dividing the eleven dimensional coordinates x^M ($M = 0, \dots, 9, 11$) into the seven dimensional flat coordinates y^i ($i = 0, \dots, 6$) and the coordinates on Taub-NUT manifold x^μ ($x^7 \equiv r$, $x^8 \equiv \theta$, $x^9 \equiv \phi$ and $x^{11} \equiv \psi$), the three form field A_{LMN} is classified into four types of fields:

$$A_{LMN} \rightarrow A_{\lambda\mu\nu}, \quad A_{i\mu\nu}, \quad A_{ij\mu}, \quad A_{ijk}.$$

Our goal in this section is to derive the $(B - F)^2$ -term and hence we are interested in $A_{i\mu\nu}$ and $A_{ij\mu}$. Since we would like to study the massless modes, we should express $A_{ij\mu}$ and $A_{i\mu\nu}$ as products of zero modes on Taub-NUT manifold and functions depending only on the flat coordinate y^i :

$$A_{ij\mu}(x^\mu, y^i) = A_\mu(x^\mu) B_{ij}(y^i), \quad (27)$$

$$A_{i\mu\nu}(x^\mu, y^i) = A_{\mu\nu}(x^\mu) V_i(y^i). \quad (28)$$

V_i becomes $U(1)$ field on the brane and B_{ij} plays the role of NS-NS two form field in the bulk.

Taub-NUT manifold has $SU(2) \times U(1)$ isometry, where $SU(2)$ represents a rotation of the base manifold R^3 and $U(1)$ corresponds to a global shift along the S^1 fiber. Therefore, we can classify modes on the manifold by two integers L and N^* . L is the angular momentum with respect to the $SU(2)$ rotation and N labels the Kaluza-Klein momentum along the S^1 fiber. In the type IIA theory context, N represents the D-particle charge, and we are interested in the modes with $N = 0$ in this paper. The vector field V_i and the pull-back B_{ij} of the two form field are scalars with respect to the $SU(2)$ rotation, so that we should consider the modes with $(L, N) = (0, 0)$. The $(0, 0)$ modes of the vector and the two form fields on Taub-NUT manifold are expressed as follows:

$$A_\mu = \left(f_1(r), 0, \frac{1}{2} f_2(r) \cos \theta, f_2(r) \right), \quad (29)$$

$$A_{\mu\nu} = \begin{pmatrix} 0 & 0 & \frac{1}{2} g_1(r) \cos \theta & g_1(r) \\ 0 & 0 & g_2(r) \sin \theta & 0 \\ -\frac{1}{2} g_1(r) \cos \theta & -g_2(r) \sin \theta & 0 & 0 \\ -g_1(r) & 0 & 0 & 0 \end{pmatrix}, \quad (30)$$

where f_1 , f_2 , g_1 and g_2 are functions depending only on r .

The vector field zero mode is defined by following equation of motion:

$$\nabla^\alpha \nabla_{[\alpha} A_{\mu]} = 0. \quad (31)$$

*To be precise, when N is odd, L must be an half integer because of the anti-periodic boundary condition due to Dirac strings with $1/2$ flux at $\theta = 0$ and π .

Note that this equation is not equivalent to $\nabla_{[\alpha} A_{\mu]} = 0$ since A_μ needs not vanish at infinity $r \rightarrow \infty$. Substituting (29) into (31), we get

$$-4m^2 f_2 + 2r(r^2 - m^2)f_2' + (r^2 - m^2)^2 f_2'' = 0, \quad (32)$$

which is easily solved to give

$$f_2 = c_1 \frac{r+m}{r-m} + c_2 \frac{r-m}{r+m}. \quad (33)$$

The function f_1 does not appear in the equation of motion. This is a reflection of the fact that f_1 is a gauge degree of freedom. Because the first term in (33) is singular at the center $r = m$, c_1 should be zero. The normalization factor c_2 should be determined so that the kinetic term of B_{ij} in the asymptotic region agrees with the third term in (13). Substituting (33) into the M-theory action (7), we get

$$\begin{aligned} -\frac{1}{2 \cdot 3!} \int d^{11}x \sqrt{g} K_{\mu i j k} K^{\mu i j k} &= -\frac{1}{2 \cdot 3!} \int dx^{11} \sqrt{g_{11,11}} \times A_\mu A^\mu \times \int d^{10}x \sqrt{g} H_{ijk} H^{ijk} \\ &= -\frac{1}{2 \cdot 3!} 2\pi l_s e^{\frac{2}{3}\phi_0} \times \frac{c_2^2}{l_s^2} e^{-\frac{4}{3}\phi_0} \times \int d^{10}x \sqrt{g} H_{ijk} H^{ijk}, \end{aligned} \quad (34)$$

which, after rescaling the metric according to (22), is reduced to

$$-\frac{1}{2 \cdot 3!} \frac{2\pi}{l_s} e^{-2\phi_0} c_2^2 \int d^{10}x \sqrt{g} H_{ijk} H^{ijk}. \quad (35)$$

Comparing this equation to (13), we can fix the constant c_2 :

$$c_2 = -\frac{1}{(2\pi)^4 l_s^{\frac{7}{2}}}. \quad (36)$$

Next, let us construct the zero mode of the two form $A_{\mu\nu}$. It should satisfy the equation of motion $\nabla^\alpha \nabla_{[\alpha} A_{\mu\nu]} = 0$, which is equivalent to

$$\nabla_{[\lambda} A_{\mu\nu]} = 0. \quad (37)$$

Because the holonomy group of Taub-NUT manifold is $SU(2)$, the zero mode of the self-dual part of $A_{\mu\nu}$, which is singlet with respect to the $SU(2)$, is a covariantly constant tensor and is not normalizable. To get a normalizable solution, we impose the anti-self-duality condition on $A_{\mu\nu}$:

$$A_{\mu\nu} = -\sqrt{-g} \epsilon_{\mu\nu\rho\sigma} A^{\rho\sigma}. \quad (38)$$

Substituting (30) into (37) and (38), we get

$$g_1(r) = -2g_2'(r), \quad g_2(r) = -\frac{r^2 - m^2}{4m} g_1(r), \quad (39)$$

and the solution is

$$g_1(r) = c \frac{4m}{(r+m)^2}, \quad g_2(r) = -c \frac{r-m}{r+m}. \quad (40)$$

We should determine the normalization factor c by the condition that the charge of the string endpoint is one [6]. An open string attached on a D6-brane is a membrane hooked on the center of the Kaluza-Klein monopole, where the S^1 fiber shrinks to a point[†]. For example, let us consider the following membrane configuration:

$$y^i = y^i(\sigma^0), \quad r = \sigma^1, \quad \psi = \sigma^2, \quad \theta, \phi = \text{const.} \quad (41)$$

This configuration represents a string attached on a D6-brane at $y^i(\sigma^0)$ and stretched along the r -direction. Using (28), the contribution of this configuration to the action (9) reads

$$S_{M2} = cg_{M2} \int dr d\psi A_{r\psi} \int dy^i V_i = 4\pi cg_{M2} \int dy^i V_i. \quad (42)$$

Therefore, c is fixed to be

$$c = \frac{1}{4\pi g_{M2}} = \frac{1}{2} \frac{1}{(2\pi)^3 l_s^{\frac{3}{2}}}. \quad (43)$$

Between these two solutions for A_μ and $A_{\mu\nu}$, the following relation is holds:

$$\nabla_\mu A_\nu - \nabla_\nu A_\mu = -\frac{1}{2\pi l_s^2} A_{\mu\nu}. \quad (44)$$

Using (27), (28) and this relation, the 4-form field strength in (7) is rewritten as

$$\begin{aligned} K_{\mu\nu ij} &= A_{\mu\nu}(\nabla_i V_j - \nabla_j V_i) + (\nabla_\mu A_\nu - \nabla_\nu A_\mu) B_{ij} \\ &= A_{\mu\nu} V_{ij}, \end{aligned} \quad (45)$$

and hence the action is

$$- \frac{1}{8} \int d^{11}x K_{ij\mu\nu} K^{ij\mu\nu} = -\frac{a}{8} \int d^7y V_{ij}^2, \quad (46)$$

with

$$a = \int d^4x (A_{\mu\nu})^2 = 32\pi^2 c^2 = 2 \frac{1}{(2\pi)^4 l_s^3}. \quad (47)$$

Rescaling the metric, we get the desired action which is identical to the second term in (5):

$$S = -\frac{1}{4(2\pi)^4 l_s^3 e^{\phi_0}} \int d^7y V_{ij}^2. \quad (48)$$

[†]This “hooking” is related to an anomalous process in which, when a D6 and a D2-brane pass through each other, a string stretched between them is created [7].

4 Chern-Simons terms

In this section we shall show that the first three terms in the Chern-Simons action (6) are reproduced from M-theory. The first term of (6) is an ordinary magnetic coupling of the R-R one form C_M . The one form field C_M is given by the eleven dimensional metric as

$$C_M = b \frac{g_{11,M}}{g_{11,11}}, \quad (49)$$

where b is a normalization factor and the electric charge is quantized in unit of b . The kinetic term of C_M is obtained from the eleven dimensional Einstein action:

$$S = -\frac{b^2}{(2\pi)^7 l_s^6} \frac{1}{4} F_{MN}^2. \quad (50)$$

Comparing the coefficient in this equation to the one in (14), the constant b is determined as

$$b = (2\pi)^{\frac{7}{2}} l_s^3. \quad (51)$$

Due to the Dirac condition, this means that the unit of magnetic charge is $2\pi/b = (2\pi)^{-\frac{5}{2}} l_s^{-3}$, and the coupling corresponding to this charge is

$$S = \frac{1}{(2\pi)^{\frac{5}{2}} l_s^3} \frac{1}{2 \cdot 7!} \int dy^{i_1} \wedge \cdots \wedge dy^{i_7} C_{i_1 \dots i_7}. \quad (52)$$

The agreement of (52) with the first term of (6) is no wonder because the coefficient of this term is determined only by the Dirac quantization condition.

Next, let us consider the term

$$- \frac{1}{2 \cdot 4!} \sqrt{-g} K_{KLMN} K^{KLMN} = - \frac{1}{2 \cdot 4! \cdot 7!} \epsilon^{ABCDEFGH IJK} K_{ABCD} \widetilde{K}_{EFGH IJK}, \quad (53)$$

in the eleven dimensional action (7). This term contains the part,

$$- \frac{1}{2 \cdot 2 \cdot 5!} \epsilon^{\mu\nu ij \alpha abcde\beta} K_{\mu\nu ij} \partial_\alpha A_{abcde\beta}, \quad (54)$$

where $K_{\mu\nu ij}$ is given by (45), and $A_{bcdef\mu}$ should be expressed as a product of the vector zero mode A_μ of the last section and the R-R 5-form C_{abcde} ,

$$A_{abcde\mu} = k C_{abcde} A_\mu. \quad (55)$$

The numerical factor k is determined so that the kinetic term of C_{abcde} is $-(2 \cdot 6!)^{-1} F_{abcde}^2$ in the asymptotic region. Substituting (55) into the kinetic term of $A_{bcdef\mu}$, we get

$$\begin{aligned} - \frac{1}{2 \cdot 7!} \int d^{11}x \sqrt{-g} \widetilde{K}_{ABCDEFGH} \widetilde{K}^{ABCDEFGH} &= - \frac{k^2}{2 \cdot 6!} \int d^{11}x \sqrt{g_{11,11}} \int d^{10}x \sqrt{-g} (F_{abcde})^2 (A_\mu)^2 \\ &= - \frac{k^2}{2 \cdot 6!} \frac{e^{-\frac{2}{3}\phi_0}}{(2\pi)^7 l_s^8} \int d^{10}x \sqrt{-g} (F_{abcde})^2, \end{aligned} \quad (56)$$

where $F_{abcdef} = \partial_a C_{bcdef} + 5$ terms. Rescaling the metric, (56) is reduced to

$$- \frac{1}{2 \cdot 6!} \frac{k^2}{(2\pi)^7 l_s^8} \int d^{10}x \sqrt{-g} (F_{abcdef})^2, \quad (57)$$

and k is determined to be

$$k = (2\pi)^{\frac{7}{2}} l_s^4. \quad (58)$$

Then, substituting (45) and (55) into (54), we get

$$\begin{aligned} S &= \frac{(2\pi)^{\frac{7}{2}} l_s^4}{2} \int d^4x \epsilon^{\alpha\beta\mu\nu} A_{\mu\nu} \partial_\alpha A_\beta \times \frac{1}{2 \cdot 5!} \int d^7y \epsilon^{abcdeij} V_{ij} C_{bcdef} \\ &= \frac{1}{(2\pi)^{\frac{3}{2}} l_s} \frac{1}{2 \cdot 5!} \int d^7y \epsilon^{abcdefg} C_{abcde} V_{fg}. \end{aligned} \quad (59)$$

Thus we have reproduced the second term of (6).

The third term in (6), CVV -term, can be extracted from the eleven dimensional Chern-Simons term (8). First, we have to determine the rescaling factor between A_{ijk} and C_{ijk} . Comparing the kinetic terms of these fields in (7) and (14), we get the relation:

$$A_{ijk} = \frac{1}{\sqrt{2\pi} l_s} C_{ijk}. \quad (60)$$

Because the three form field in seven dimension A_{ijk} is a scalar on Taub-NUT, its zero mode is a constant which does not depend on x^μ . Replacing the A_{LMN} in (8) with (60) and the two K_{ABCD} with (45), we get exactly the third term of (6):

$$\begin{aligned} S &= -\frac{\sqrt{2\kappa_{11}^2}}{\sqrt{2\pi} l_s} \times \frac{1}{4} \int d^4x \epsilon^{\mu\nu\rho\sigma} A_{\mu\nu} A_{\rho\sigma} \times \frac{1}{3! \cdot 2 \cdot 2 \cdot 2} \int d^7y \epsilon^{ijklmnp} C_{ijk} V_{lm} V_{np} \\ &= \frac{l_s}{\sqrt{2\pi}} \frac{1}{3! \cdot 2 \cdot 2 \cdot 2} \int d^7y \epsilon^{ijklmnp} C_{ijk} V_{lm} V_{np}. \end{aligned} \quad (61)$$

5 Discussion

In this paper, we have shown that the V^2 term and the $C + CV + CVV$ terms are correctly reproduced from the Kaluza-Klein monopole. We shall give some comments about the terms we have not discussed in this paper.

The Born-Infeld action (5) expanded to quadratic order contains another term $(\partial\phi^a)^2$ where ϕ^a are scalar fields on D6-brane and represent the transversal shifts of the brane. These fields belong to the **3** representation of the isometry group $SU(2)$. The shift of the center

of the Kaluza-Klein monopole implies that the metric varies. Therefore, the $(L = 1, N = 0)$ mode of the metric would be relevant for ϕ^a .

Another term we have not discussed is the $CVVV$ -term in (6). This is cubic order in the vector field V_i , which is a part of the three form field A_{LMN} in eleven dimension. The unique cubic term of the three form field in the action is (8). This term does not contain the metric and, apparently, it seems impossible to extract the $CVVV$ -term from this term. However, the anti-self-duality condition (38) of $A_{\mu\nu}$ depends on the metric and the AKK -term can contain C_μ indirectly, and hence the $CVVV$ -term could be reproduced from the AKK -term. To examine this possibility, we have to solve the zero mode equations by taking account of the variation of the background metric.

I would like to thank H. Hata for valuable discussions and careful reading of the manuscript.

Note Added: After this work was completed, the paper [8] appeared, which also proposes the Born-Infeld effective action of Kaluza-Klein monopole by a different approach from ours.

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